$$dv = \left(\frac{c\dot{m}}{m}\right) dt$$

$$(\text{右辺}) = \frac{c\dot{m}}{m} dt = \frac{c\dot{m}}{m_0 - \dot{m}t} dt$$

全備質量 → m_0

m_p：推進薬質量

t_b：燃焼時間

$$= \frac{c\left(\frac{m_p}{t_b}\right)}{m_0 - \left(\frac{m_p}{t_b}\right)t} dt = \frac{c\left(\frac{m_p}{t_b}\right)\left(\frac{m_0}{m_0}\right)}{m_0\left(1 - \frac{m_p}{m_0}\frac{t}{t_b}\right)} dt$$

$$= \frac{\cancel{m_0}\frac{c}{t_b}\left(\frac{m_p}{m_0}\right)}{\cancel{m_0}\left(1 - \frac{m_p}{m_0}\frac{t}{t_b}\right)} dt = \frac{\frac{c}{t_b}\left(\frac{m_p}{m_0}\right)}{1 - \frac{m_p}{m_0}\frac{t}{t_b}} dt$$

空虚質量 → m_f

$$\frac{m_p}{m_0} = \frac{m_0 - m_f}{m_0} = 1 - \frac{m_f}{m_0} = 1 - MR$$

$\frac{m_f}{m_0}$ → 質量比

$$= \frac{\frac{c}{t_b}(1-MR)dt}{1 - (1-MR)\frac{t}{t_b}}$$

よって

$$\int_0^{t_b} dv = \int_0^{t_b} \frac{\frac{c}{t_b}(1-MR)}{1-(1-MR)\frac{t}{t_b}}\,dt$$

$$(\text{左辺}) = \left[v\right]_0^{t_b} = v_f - v_0$$

$$(\text{右辺}) = C\int_0^{t_b} \frac{\frac{1}{t_b}(1-MR)}{1-(1-MR)\frac{t}{t_b}}\,dt$$

$$= C\int_0^{t_b} \frac{1-MR}{t_b-(1-MR)t}\,dt$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)|+C$$

$$= C\left[-\ln\{t_b-(1-MR)t\}\right]_0^{t_b}$$

$$= -C\left[\ln\{t_b-(1-MR)t_b\} - \ln\{t_b-(1-MR)0\}\right]$$

$$= -C\left[\ln(t_b - t_b + MR\,t_b) - \ln t_b\right]$$

$$= -C\{\ln(MRt_b) - \ln t_b\}$$

$$= -C\ln\frac{MRt_b}{t_b} = -C\ln MR$$

$$= C\ln\frac{1}{MR}$$

$\log_a b = -\log_a \frac{1}{b}$

したがって

$$v_f - v_0 = C\ln\frac{1}{MR}$$

ロケットの初速 $v_0 = 0$ とすると

$$v_f = C\ln\frac{1}{MR}$$